GRADE : 12

SUBJECT : Mathematics

TITLE : June Paper 1

EXAMINER : Mr A. Slaughter

TOTAL MARKS : 150

DATE : 3 / 6 /20 16

SOLUTIONS

TIME : 3 hour(s)

1.1.1.	$x^2 = 5x$	
	$x^2 - 5x = 0$ ✓	
	$x(x-5) = 0$ ✓	
	$\therefore x = 0$ or 5 ✓	3
1.1.2.	$3x^2 - 4x - 12 = 0$	
	$(\quad)(\quad) = 0$ xx	
	✓ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-12)}}{2(3)}$	
	$= \frac{4 \pm \sqrt{160}}{6}$	
	$= 2{,}77$ ✓ or $-1{,}44$ ✓	3
1.1.3.	$10x^{-2/3} + 8x^{-4/3} = 3$	
	$8x^{-\frac{4}{3}} + 10x^{-\frac{2}{3}} - 3 = 0$	
	$(4x^{-2/3} - 1)(2x^{-2/3} + 3) = 0$ ✓	
	$\therefore x^{-2/3} = \frac{1}{4}$ ✓ or $x^{-2/3} = -\frac{3}{2}$	
	$(x^{-2/3})^{-3/2} = \pm\left(\frac{1}{4}\right)^{-3/2}$ ✓ ✓ no soln ✓	
	$x = \pm 8$ ✓	6

1.1.4.	$0 \le -x(6x+5) + 4$	
	$0 \le -6x^2 - 5x + 4$	
	$6x^2 + 5x - 4 \le 0$ ✓	
	$(2x-1)(3x+4) \le 0$ ✓	
	$+ \quad 0 \quad - \quad 0 \quad +$; $-4/3$, $1/2$	
	$\therefore -\frac{4}{3} \le x \le \frac{1}{2}$ ✓	3
1.1.5.	$\frac{\sqrt{x}\,(2-x)}{2^x(x-1)} \geq 0$	
	$\mathbb{R}'$ $\quad 0 \quad - \quad$ UD $\quad + \quad 0 \quad -$; 0 , 1 , 2	
	$\therefore x = 0$ ✓ or $1 < x \le 2$ ✓	2
	no penalty if no "or"	
1.2.	$6x^2 - 3y = 11x + 10$	
	$\frac{1}{3}x - y = \frac{16}{3}$	
	$\frac{1}{3}x - \frac{16}{3} = y$ ✓	

	$6x^2 - 3\left(\frac{1}{3}x - \frac{16}{3}\right) = 11x + 10$		1.3.	$\dfrac{2^{2015}}{2^{2017} - 3 \cdot 2^{2012}}$	
	$6x^2 - x + 16 = 11x + 10$				
	$6x^2 - 12x + 6 = 0$			$= \dfrac{2^{2015}}{2^{2012} \cdot 2^5 - 3 \cdot 2^{2012}}$	
	$\div 6: \quad x^2 - 2x + 1 = 0$				
	$(x-1)(x-1) = 0$			$= \dfrac{2^{2015}}{2^{2012}(2^5 - 3)}$	
	$\therefore x = 1$			cf	
	$\therefore y = \frac{1}{3}(1) - \frac{16}{3}$			$= \dfrac{2^3}{32 - 3}$	
	$= -5$			$= \dfrac{8}{29}$	3
	$\therefore x = 1$ and $y = -5$	6			
			1.4.	$\Delta = 21 - 4k$	
1.2.2.	$6x^2 - 3y = 11x + 10$			For real roots	
	$6x^2 - 11x - 10 = 3y$			$\Delta \geq 0$	
	$2x^2 - \frac{11}{3}x - \frac{10}{3} = y$			$21 - 4k \geq 0$	
	parabola			$k \leq \frac{21}{4}$ 5,25	
	$\frac{1}{3}x - y = \frac{16}{3}$			k | Δ	
	$\frac{1}{3}x - \frac{16}{3} = y$			0 | 21	
	straight line			1 | 17	
	pts			2 | 13	
	The parabola and str			3 | 9	
	line are tangential			4 | 5	
	at $(1; -5)$ as they			5 | 1	
	only intersect in			For rational roots	
	one point.	2		Δ = perfect square	
				$\therefore k = 3$ and 5	2

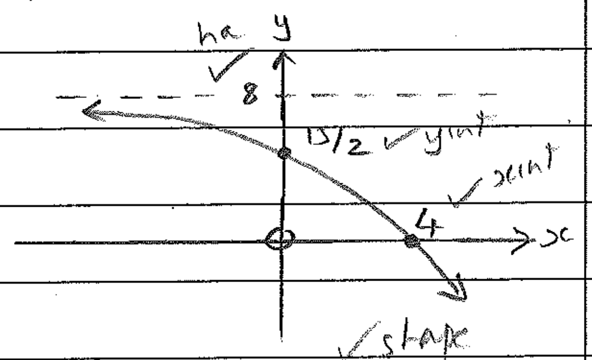
-1 each incorrect answer

1.5.	$y = x + \frac{1}{x}$	
	$x \in \mathbb{R}$ ie real	
	$x \neq 0$	
1.5.1.	LCD $= x$	
	($\therefore x \neq 0$)	
	x thru	
	$xy = x^2 + 1$	
	$0 = x^2 - yx + 1$ ✓ std order	1
1.5.2.	$\Delta = (-y)^2 - 4(1)(1)$ ✓	
	$= y^2 - 4$ ✓	2
1.5.3.	Since x is real	
	$\Delta \geqslant 0$	
	$y^2 - 4 \geqslant 0$ ✓	
	$(y-2)(y+2) \geqslant 0$ ✓	
	$+ \quad -2 \quad - \quad 2 \quad +$	
	$\therefore y \leq -2$ or $2 \leq y$ ✓	3

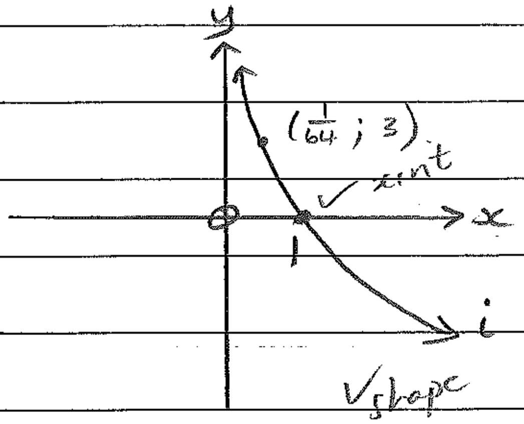
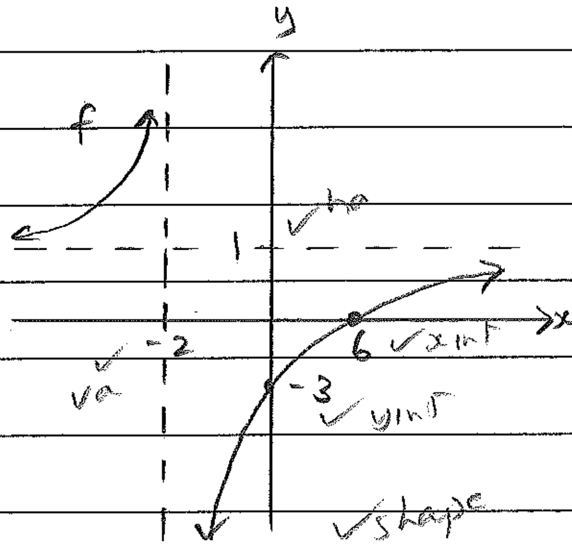
2.1.	T_{10} ; T_{11} ; T_{12}	
	$2x+3$; $4x+10$; $10x-3$	
2.1.1.	Arithmetic,	
	$\therefore 4x+10-(2x+3) = 10x-3-(4x+10)$ ✓	
	$4x+10-2x-3 = 10x-3-4x-10$	
	$2x+7 = 6x-13$	
	$20 = 4x$ ✓	
	$5 = x$	2
2.1.2.	$T_{10} = 2(5)+3 = 13$ ✓	
	$T_{11} = 4(5)+10 = 30$ ✓	2
2.1.3.	$d = 30-13 = 17$ ✓d	
	$T_{10} = 13$	
	$a+9d = 13$	
	$a+9(17) = 13$	
	$a = -140$ ✓a	
	$\therefore T_{500} = a+499d$ ✓f	
	$= -140 + 499(17)$	
	$= 8343$ ✓	4

2.2.	$\sum_{k=7}^{95}(3-5k)$		
	$= -32 - 37 - 42 \ldots$		
	$a = -32$ ✓ $d = -5$ ✓		
	$n = 95 - 7 + 1$		
	$= 89$ ✓		
	$S_n = \frac{n(2a+(n-1)d)}{2}$		
	$S_{89} = \frac{89(2(-32)+(88)(-5))}{2}$ ✓		
	$= -22\ 428$ ✓ answ only $0/5$		5
2.3.	$\frac{3}{4} + \frac{1}{2} + \frac{1}{3} + \ldots$		
2.3.1.	$a = \frac{3}{4}$ $r = \frac{2}{3}$ ✓r		
	$S_\infty = \frac{a}{1-r}$		
	$= \frac{3/4}{1-2/3}$ ✓f+s		
	$= \frac{9}{4}$		
	$S_n = \frac{a(r^n-1)}{r-1}$		
	$= \frac{3/4((\frac{2}{3})^n - 1)}{2/3 - 1}$ ✓f+s		
	$= -\frac{9}{4}((\frac{2}{3})^n - 1)$		
	$= -\frac{9}{4}(\frac{2}{3})^n + \frac{9}{4}$		
	$\therefore S_\infty - S_n = \frac{1024}{59049}$		
✓ set up	$\frac{9}{4} - (-\frac{9}{4}(\frac{2}{3})^n + \frac{9}{4}) = \frac{1024}{59049}$		
	$\frac{9}{4} + \frac{9}{4}(\frac{2}{3})^n - \frac{9}{4} = \frac{1024}{59049}$		
	$\frac{9}{4}(\frac{2}{3})^n = \frac{1024}{59049}$		
	$(\frac{2}{3})^n =$ ✓ $0,0077\ldots$		

	$n = \frac{\log(0,0077\ldots)}{\log(2/3)}$ ✓	
	$= 12$ ✓	7
2.3.2	h_{max}	
	$= \overbrace{1,5}^{✓} + S_\infty$	
	$= 1,5 + \frac{9}{4}$ (2.3.1)	
	$= \frac{15}{4}$ m ✓ 3,75	2
	don't penalise units	
2.4.	$\frac{4}{16}; \frac{-4}{8}; \frac{-18}{4}; \frac{-38}{2}$	
	• den : 16; 8; 4; 2; ...	
	$a = 16$ $r = \frac{1}{2}$	
	$\therefore T_n^{den} = ar^{n-1}$	
	$= 16(\frac{1}{2})^{n-1}$ ✓	
	• num 4; −4; −18; −38	
	−8 −14 −20	
	−6 −6	
	$d_2 = 2a$ $d_1 = 3a+b$ $T_1 = a+b+c$	
	$-6 = 2a$ $-8 = 3(-3)+b$ $4 = -3+1+c$	
	$-3 = a$ ✓ $1 = b$ ✓ $6 = c$ ✓	
	$\therefore T_n^{num} = -3n^2 + n + 6$ ✓	
	So,	
	$T_n = \frac{-3n^2+n+6}{16(\frac{1}{2})^{n-1}}$ ✓÷	6

3.1.	$f(x) = -2^{x-1} + 8$	
	$y = -1 . 2^{x-1} + 8$	
3.1.1.	exponential	
	· y int : $y = -2^{0-1} + 8$	
	$= \frac{15}{2}$ 7,5	
	· x int : $0 = -2^{x-1} + 8$	
	$2^{x-1} = 2^3$	
	$x - 1 = 3$	
	$x = 4$	
	· ha : $y = 8$	
		4
3.1.2.	$y \in (-\infty ; 8)$ ✓	1
3.1.3.	$x = -1$: $y = -2^{-1-1} + 8$	
	$= \frac{31}{4}$	
	$\therefore (-1 ; \frac{31}{4})$ ✓	
	$x = 1$: $y = -2^{1-1} + 8$	
	$= 7$	

	$\therefore (1 ; 7)$ ✓	
	So, av grad $= \frac{\Delta y}{\Delta x}$	
	$= \frac{7 - 31/4}{1 - (-1)}$	
	$= -\frac{3}{8}$ ✓	3
3.1.4.	Refl x axis	
	· $x \to x$	
	· $y \to -y$	
	$y = -2^{x-1} + 8$	
	$-y = -2^{x-1} + 8$ ✓	
	$y = 2^{x-1} - 8$ ✓	2
	answer only 2/2	
3.1.5.	$h(x) = -16 . 2^{x-1} + 10$	
	$= -2^4 . 2^{x-1} + 10$	
	$= -2^{x-1+4} + 10$	
	$f(x) = -2^{x-1} + 8$	
	So	
	✓ · 2 units upwards ↑	
	· 4 units left ← ✓ ✓	3

3.2.	$i(x) = \log_{\frac{1}{4}} x$			LCD $= x+2$	
				$(\therefore x \neq -2)$	
3.2.1.	 	2		x thru $8 = x+2$ $6 = x$ · ha: $y = 1$ · va: $x = -2$ · shape: $a = -8$	
3.2.2.	$\log_{\frac{1}{4}} x = 3$ $(\frac{1}{4})^3 = x$ ✓ $\frac{1}{64} = x$ ✓	2		 	5
3.2.3.	$\log_{\frac{1}{4}} x \geq 3$ $y_i \geq 3$ $\therefore x \in (0; \frac{1}{64}]$ interval ✓ not^n ✓	2			
4.1.	$f(x) = -\frac{8}{x+2} + 1$		4.1.2.	a : $-8 \rightarrow 8$ $\therefore$	
4.1.1	· hyperbola · yint: $y = -\frac{8}{0+2} + 1$ $= -3$			$\therefore x = -2$ ✓ or $y = 1$ ✓	2
	· xint: $0 = -\frac{8}{x+2} + 1$ $\frac{8}{x+2} = 1$		4.1.3.	$5 \leftarrow \therefore x \rightarrow x+5$ $\therefore y = -\frac{8}{x+7} + 1$ ✓ whole eqn perfect	1

Detach this page from your question paper and staple it, in order, with your foolscap answers.

Name and Surname : ....

ANSWER PAGE FOR QUESTION 5

5.1.

$y = a(x+4)(x-10)$ ✓ ✓

$= a(x^2 - 6x - 40)$ ✓ x out

but $a = -2$

$\therefore$

$y = -2(x^2 - 6x - 40)$

$= -2x^2 + 12x + 80$

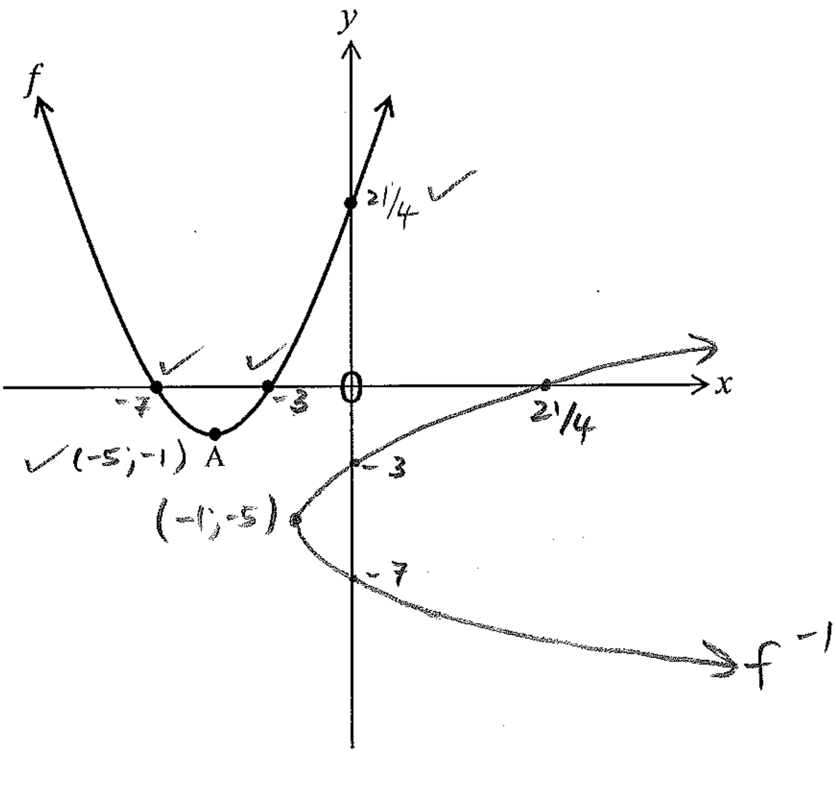
$\therefore$ $b = 12$ $c = 80$

✓ both

-4 10 $x = 3$ $\underbrace{\quad}_{7}$ $\underbrace{\quad}_{7}$

4

5.2.



4

5.2. 1.	$y = \frac{1}{4}(x+5)^2 - 1$		
	y int: $y = \frac{21}{4}$ 5,25 tp: $(-5;-1)$		
	x int: $0 = \frac{1}{4}(x+5)^2 - 1$		
	$1 = \frac{1}{4}(x+5)^2$		
	$4 = (x+5)^2$		
	$\pm 2 = x+5$		
	$-5 \pm 2 = x$		
	-7 or $-3 =$		
2.	See sketch		
	✓ tp		
	✓ x and y int's	3	
	✓ shape		
3.	$f: \quad y = \frac{1}{4}(x+5)^2 - 1$		
	$f^{-1}: \quad x = \frac{1}{4}(y+5)^2 - 1$ ✓		
	$x+1 = \frac{1}{4}(y+5)^2$		
	$\times 4: \quad 4x+4 = (y+5)^2$ ✓		
	$\pm\sqrt{4x+4} = y+5$ ✓ $\pm\sqrt{\ }$		
	$-5 \pm \sqrt{4x+4} = y$ ✓		
	$-5 \pm \sqrt{4(x+1)} =$		
	$-5 \pm 2\sqrt{x+1} =$	4	
4.1.	f is a many to one, and not ✓	1	
	a one to one, function.		
4.2.	$x \geq -5$ ✓ or $x \leq -5$	1	
	any one		

4.2.	$y = \frac{5}{x-4} + q$ ✓	
	AOS :	
	$y = -(x-4) + q$	
	$= -x + 4 + q$	
	$y = -x + 7$	
	$\therefore \ 4 + q = 7$	
	$q = 3$ ✓	
	answer only $^2/_2$	2
5.	See answer sheet	
6.1.	$A = P(1+i)^n$	
	$2x = x\left(1 + \frac{6}{1200}\right)^n$ ✓✓	
	$\div x$ (as $x \neq 0$)	
	$2 = \left(\frac{201}{200}\right)^n$	
	$n = \frac{\log 2}{\log(201/200)}$ ✓	
	$= 138,97 \ldots$ months	
	$= 11,58 \ldots$ years	
	$\therefore$ 12 full years ✓	4

6.2.	$A = P(1-i)^n$	
	$x - 30\,000 = x\left(1 - \frac{7}{100}\right)^5$ ✓✓	
	$= x \cdot 0,69\ldots$	
	$x - 0,69\ldots x = 30\,000$ ✓	
	$0,30\ldots x = 30\,000$	
	$x = R\,98\,583,15$ ✓	4
6.3.	$\left(1 + \frac{i_{nom}}{k}\right)^k = 1 + i_{ea}$	
	$\left(1 + \frac{6}{1200}\right)^{12} = 1 + i_{ea}$	
	$\left(1 + \frac{i_{nom}}{200}\right)^2 = 1 + i_{ea}$	
	$\therefore \left(1 + \frac{6}{1200}\right)^{12} = \left(1 + \frac{i_{nom}}{2}\right)^2$ ✓✓	
	$1,06\ldots = \left(1 + \frac{i_{nom}}{2}\right)^2$	
	$\sqrt{1,06\ldots} = 1 + \frac{i_{nom}}{2}$ ✓	
	$1,03\ldots = 1 + \frac{i_{nom}}{2}$	
	$0,060\ldots = i_{nom}$	
	$6,08\ \% = I_{nom}$	
	$\therefore$ 6,08 % pa comp half yearly.	

7.1.	$R = f(-3)$	
	$-14 = -2(-3)^3 + a(-3)^2 + 4$ sub $x=-3$	
	$-72 = 9a$	
	$-8 = a$	3
7.2.	1. $f\left(-\frac{5}{3}\right)$	
	$= 6\left(-\frac{5}{3}\right)^3 - 2\left(-\frac{5}{3}\right)^2 + \left(-\frac{5}{3}\right) + 35$	
	$= 0$	
	$\therefore$ $3x+5$ is a factor	2
7.2.	2. $6x^2 - 2x^2 + x + 35$	
	$= (3x+5)(2x^2 + bx + 7)$	
	$10x^2$, $3bx^2$	
	$= -2x^2$	
	$10x^2 + 3bx^2 = -2x^2$	
	$3bx^2 = -12x^2$	
	$3b = -12$	
	$b = -4$	
	$\therefore (2x^2 - 4x + 7)$	3

8.1.	$f(x) = \frac{3}{x} - 1$	
	$f(x+h) = \frac{3}{x+h} - 1$	
	$f'(x)$	
	$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ f	
	$= \lim_{h\to 0} \frac{\frac{3}{x+h} - 1 - \left(\frac{3}{x} - 1\right)}{h}$	
	$= \lim_{h\to 0} \frac{\frac{3}{x+h} - 1 - \frac{3}{x} + 1}{h}$	
	$= \lim_{h\to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h}$	
	$= \lim_{h\to 0} \frac{3x - 3(x+h)}{x(x+h)} \div h$	
	$= \lim_{h\to 0} \frac{3x - 3x - 3h}{x(x+h)} \times \frac{1}{h}$	
	$= \lim_{h\to 0} \frac{-3h}{x(x+h)} \times \frac{1}{h}$	
	$= \lim_{h\to 0} \frac{-3}{x(x+h)}$	
	$= \frac{-3}{x(x+0)}$	
	$= -\frac{3}{x^2}$	6
8.2.	1. $y = \frac{x^2 + 5}{4\sqrt[3]{x}}$	
	$= \frac{x^2 + 5}{4x^{1/3}}$	
	$= \frac{x^2}{4x^{1/3}} + \frac{5}{4x^{1/3}}$	
	$= \frac{1}{4}x^{5/3} + \frac{5}{4}x^{-\frac{1}{3}}$	
	$\frac{dy}{dx} = \frac{5}{12}x^{2/3} - \frac{5}{12}x^{-4/3}$	4

8.2.2. $f(x) = x^{\frac{1}{2}}\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)$

$= x - x^{0}$

$= x - 1$ ✓ ✓

$\therefore f'(x) = 1$ ✓ — 3

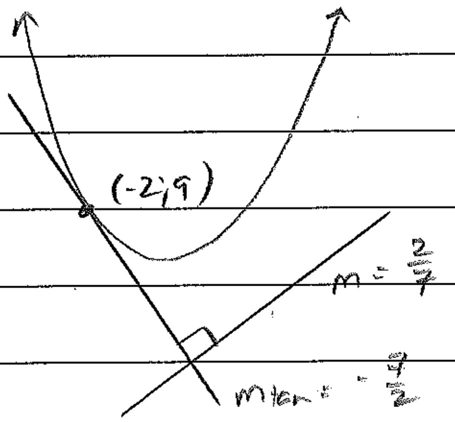
8.2.3. $D_x\left[\frac{8x^3 - 27}{2x - 3}\right]$

$= D_x\left[\frac{(2x-3)(4x^2+6x+9)}{2x-3}\right]$

$= D_x\left[4x^2 + 6x + 9\right]$ ✓

$= 8x + 6$ ✓ — 2

8.3.



POC : $y = ax^2 + bx + 5$

sub $(-2;9)$

✓ $9 = a(-2)^2 + b(-2) + 5$

$4 = 4a - 2b$

$\div 2$: $2 = 2a - b$

grad :

$7y - 2x + 21 = 0$

$7y = 2x - 21$

$y = \frac{2}{7}x - 3$

$\therefore m_{tan} = -\frac{7}{2}$ $\perp$

$m_{tan} = f'(x)$

$-\frac{7}{2} = 2ax + b$

✓ $-\frac{7}{2} = 2a(-2) + b$ ✓

$-\frac{7}{2} = -4a + b$

$b = 2a - 2$

$-\frac{7}{2} = -4a + (2a - 2)$

$-\frac{7}{2} = -4a + 2a - 2$

$2a = \frac{3}{2}$

$a = \frac{3}{4}$ ✓

$\therefore b = 2\left(\frac{3}{4}\right) - 2$

$b = -\frac{1}{2}$ ✓

CA sim eqns mode 5-1 — 5